Black hole fermionic radiance and D brane decay

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The semiclassical grey-body factor for massless fermion emission from the four dimensional black hole described by an ensemble of intersecting triplets of D- five-branes is shown to be consistent with the (statistical) decay rate of the branes (in the 'long' D-string approximation) into massless fermionic closed string states, subject to assumptions regarding the energy distribution of colliding open string states.

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Entropies of almost ideal gases of specific BPS and 'near'-BPS D brane configurations are known to match the Bekenstein-Hawking entropies [1] of corresponding black hole solutions around their extremal limit. The instability of (a gas of) excited (non-BPS) D branes owing to decay into closed string modes also quantitatively simulates, [2] in the 'long' string [3] *and* 'dilute gas' [10] approximation, Hawking evaporation and absorption rate [4] of the corresponding black hole. The observed similarities, however, do not yet appear to illumine crucial issues of generic black hole physics like the causal structure associated with horizon formation, or the information loss problem [5]. Also, disagreements of the rates were observed in few cases [6]. Therefore, their scope warrants careful assessment, more so because attention thus far has been exclusively confined (with a very recent exception [7]) to emission of bosons [2,3,5,6,8–10]. The present letter focuses on Hawking radiation of a specific four dimensional black hole into massless neutral *fermions*, in relation to the (statistical) decay of corresponding D-brane configurations into massless fermionic closed string states.

We consider the 'long' D-string which happens to be the line of intersection of three intersecting five-branes in M-theory [9] toroidally compactified to four spacetime dimensions. A four dimensional black hole solution to the low energy effective theory is given by the metric

$$\begin{aligned} ds^2 &= -f^{-1/2} h dt^2 + f^{1/2}\left(h^{-1}dr^2 + r^2 d\Omega^2\right) , \\ f &= \prod_{i=1}^{4}\left(1 + \frac{r_i}{r}\right), \quad h = 1 - \frac{r_0}{r} , \end{aligned} \tag{1}$$

where, r_i are related to the four $U(1)$ charges of the black hole, in turn proportional respectively to the numbers of the three five-branes under consideration (for $i = 1, 2, 3$), and the energy-momentum along the intersection line D-string. The quantity r_0 is the location of the horizon. An almost ideal (or dilute) gas of these D-brane configurations corresponds to the domain r_0 , $r_4 \ll r_i$, $i = 1, 2, 3$ [10]. We shall henceforth confine to this domain. The semiclassical greybody factor for massless (chiral) fermion emission from this black hole is calculated from the solutions to the Weyl equation in the background (1), in the 'near' and the 'far' regions with respect to the black hole horizon, and subsequently matching these solutions at some intermediate region to determine the incoming and absorbed fluxes. The ratio of these fluxes yields the required absorption cross section (grey-body factor) [10,11].

A convenient choice for the local tetrad components appropriate to (1) is given by

$$\begin{aligned} e^0_t &= f^{-1/4}h^{1/2} \ , \ e^3_r = f^{1/4}h^{-1/2}, \\ e^1_\theta &= f^{1/4} r \ , \ e^2_\phi = f^{1/4}\, r\sin\theta \ , \end{aligned} \tag{2}$$

yielding the Weyl equations (for the radial components of the Weyl spinor field, assumed left-handed)

$$\begin{aligned} i\, r\sqrt{f/h}\,\omega\, R_1 + r\sqrt{h}\,\frac{dR_1}{dr} &= -\lambda\, R_2 \ , \\ i\, r\sqrt{f/h}\,\omega\, R_2 - r\sqrt{h}\,\frac{dR_2}{dr} &= \lambda\, R_1 \ , \end{aligned} \tag{3}$$

where λ is a constant, and ω is the frequency associated with the time dependence of the solutions, assumed $\exp(-i\omega t)$.

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Near the horizon ($r \to r_0$), we introduce the variable $z \equiv 1 - r_0/r$ and approximate f as

$$f \;=\; K \;\left[(1-z)^4 \;\sinh^2\sigma + (1-z)^3\right] \;,$$

where $K \equiv r_1 r_2 r_3/r_0^3$, and $r_4 = r_0 \sinh^2\sigma$ defines σ. From eq. (3), a second order differential equation is easily deduced for either of the radial functions. For R_1, we take the ansatz

$$R_1 \;=\; A\; z^m \;(1-z)^n \;F(z) \;,$$

and restricting ourselves to the region $r \sim r_0$, we obtain a hypergeometric equation for $F(z)$

$$z\,(1-z)\;\frac{d^2F}{dz^2} \;+\; \left[\left(2m+\frac{1}{2}\right) - z\left(1+2m+2n+\frac{1}{2}\right)\right]\;\frac{dF}{dz}$$
$$-\left[\left(m+n+\frac{1}{4}\right)^2 \;+\; \mu(\omega,\sigma)\right] F \;=\; 0 \;. \tag{4}$$

Here, we have already made the choices, $m = -i\left(\frac{a+b}{2}\right)$, $n^2 = \lambda^2$ with $a(resp.\, b) \equiv \omega\sqrt{r_1 r_2 r_3/r_0}\; e^{\sigma}\; (resp.\; e^{-\sigma})$. In the regime $a \approx b$ corresponding to $\sigma \sim 0$, the function $\mu(\omega,\sigma) = -1/16 + \imath(a+b)/8$ and hypergeometric function relevant to (4) is given by $F(\alpha,\beta,\gamma;z)$ where, $\alpha \approx -i\frac{3}{4}(a+b)+n+\frac{1}{2}$, $\beta \approx -i\frac{1}{4}(a+b)+n$, $\gamma = -i(a+b)+\frac{1}{2}$. The physical significance of this regime may be open to question since it corresponds to $r_4 \ll r_0$, and since all semiclassical considerations pertain to behaviour outside the blackhole horizon. However, we include this for completeness.

In the other regime where $\sigma \geq 1$ corresponding to $a > b$, we have $\mu(\omega,\sigma) = -\left[1/4 - \imath(a-b)/2\right]^2$. The solution for $R_1(z)$ in the near zone can thus be shown to be

$$R_1^{near} = A z^{-i(a+b)/2}\;(1-z)^n\;F\left(\alpha,\beta,\gamma;z\right)\;,\;\alpha = -ia+n+\frac{1}{2},\;\beta = -ib+n,\;\gamma = -i\left(a+b\right)+\frac{1}{2}\;. \tag{5}$$

For large σ, $b \approx 0$, and parameters are $\alpha = -ia+n+\frac{1}{2}$, $\beta = n$, $\gamma = -ia+\frac{1}{2}$. This can be derived alternatively by using the Newman-Penrose formalism. In either case, in the limit $z \to 0$, $R_1^{hor}(z) = Az^{-i(a+b)/2}$.

The near zone solutions obtained above are to be matched by extrapolation to the regime $z \to 1$ to the small distance limit of the solution in the far zone ($f \to 1$, $h \to 1$). The latter solution can be shown to be given in terms of the Whittaker function [12] as

$$R_1^{far}(r) \;=\; \frac{B}{\sqrt{\omega r}}\;W_{\frac{1}{2},n}(\omega r)\;. \tag{6}$$

Using the small distance limit of the Whittaker function [12], and matching with the near solutions yields the ratio of the constants A and B, in absolute value, to be

$$\left|\frac{B}{A}\right| \;=\; \frac{1}{\sqrt{2}}(2\omega r_0)^n\left|\frac{\Gamma(n)}{\Gamma(2n)}\frac{\Gamma(\gamma)\;\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\;\Gamma(\gamma-\beta)}\right| \;. \tag{7}$$

Now, with the conserved fermionic flux given by

$$\mathcal{F} \;=\; |R_1|^2 - |R_2|^2 \;, \tag{8}$$

the absorption cross section of interest[1] can be expressed as

$$\sigma_{abs} \;=\; \frac{2\pi}{\omega^2}\;\frac{\mathcal{F}_0}{\mathcal{F}_\infty} \;=\; \frac{\pi}{\omega^2}\;\left|\frac{A}{B}\right|^2 \;, \tag{9}$$

where, $\mathcal{F}_0$ $(resp.\mathcal{F}_\infty)$ is the flux entering the black hole horizon (resp. flux arriving from past infinity). Thus, the Hawking radiation rate for fermions is given by

[1]The contribution of R_2 is negligible.

$$\Gamma_H \;=\; \frac{\pi}{\omega^2}\;\left|\frac{A}{B}\right|^2\;(e^{2\pi(a+b)}+1)^{-1}\;\frac{d^3k}{(2\pi)^3}\;. \tag{10}$$

For $a \approx b$, using eq. (7) and appropriate values of the parameters $\alpha,\ \beta\ and\ \gamma$ with $n=-1$ (which corresponds to s-wave solution for the Weyl fermion) we get

$$\Gamma_H \;=\; \frac{1}{4}\pi^2 r_0^2(a+b)[\frac{1}{4}+\frac{9}{16}(a+b)^2]\;\left(e^{\frac{\pi}{2}(a+b)}-1\right)^{-1}\left(e^{\frac{3}{2}\pi(a+b)}+1\right)^{-1}\;\frac{d^3k}{(2\pi)^3} \tag{11}$$

For $a > b$, we get, for $n=-1$,

$$\Gamma_H \;=\; 4\pi^2 r_0^2\frac{b(\frac{1}{4}+a^2)}{(e^{2\pi a}+1)\;(e^{2\pi b}-1)}\frac{d^3k}{(2\pi)^3}\;. \tag{12}$$

Also, with $n=-1$ one obtains for $a \gg b$

$$\Gamma_H \;=\; 2\pi r_0^2\;\left[\frac{1}{4}\;+\;a^2\right]\;\left(e^{2\pi a}\;+\;1\right)^{-1}\;\frac{d^3k}{(2\pi)^3}, \tag{13}$$

which essentially is the solution found in ref. [7]. This result can be derived using the alternative Newman-Penrose formalism. Also, taking $\omega \to 0$, we reproduce the general result in [13].

Massless closed string fermionic modes (gravitinos) are emitted from the decay of excited intersection D-strings (corresponding to the black hole considered above) via collision of bosonic and fermionic massless open string modes propagating on them. Gravitinos with vector polarization in the compact direction emerge as chiral spin-1/2 particles akin to those considered above. The calculation of the D-string decay rate into such particles to lowest order in string coupling entails evaluating the disc amplitude with a fermionic and a bosonic vertex operator inserted at the boundary and a closed string fermionic vertex operator in the bulk. Since the latter can be decomposed into open string bosonic and fermionic vertex operators, the requisite matrix element may be written, with suitable identifications [14], as (in the light cone Green Schwarz formalism [15])

$$\mathcal{A}(p_1,p_2,p_3,p_4)\;=\;V_{CKG}^{-1}\int\prod_{i=1}^{4}dz_i\;\;< V_F(1)\;V_B(2)\;[V_B(3)\;V_F(4)\;+\;V_B(4)\;V_F(3)] >\;\;, \tag{14}$$

where,

$$\begin{aligned} &V_B(n) \equiv V_B(\zeta_n,p_n,z_n) \equiv \zeta_n^I B_n^I(p_n,z_n)\;e^{ip_n\cdot X(z_n)} \\ &V_F(n) \equiv V_F(u_n,p_n,z_n) \equiv \left(u^a F^a(p_n,z_n)+u^{\dot a}F^{\dot a}(p_n,z_n)\right)e^{p_n.X(z_n)}\;, \end{aligned} \tag{15}$$

with $n=1,\ldots,4\ I,a,\dot a=1,\ldots,8$. In this formulation, matrices M^{IJ} and $\mathcal{M}^{ab}$, $\mathcal{M}^{\dot a\dot b}$ have been introduced a la' [15] to account for the boundary conditions associated with the D-string under consideration.

Rather than explicitly computing the rhs in (14), one can take recourse to the Ward identities corresponding to the surviving spacetime supersymmetries for the D-string [15], which may be expressed schematically as

$$V_{CKG}^{-1}\int\prod_{i=1}^{4}dz_i\;\;< [\eta Q\;,\;V_F(1)V_B(2)V_B(3)V_B(4)] >\;\;=\;0\;. \tag{16}$$

This enables us to express the desired amplitude purely in terms of the bosonic amplitude calculated for instance in ref. [16]. Here we merely estimate the energy dependence of the amplitude to check consistency with the semiclassical results. The amplitude for two open string bosons on the D-string fusing into a graviton with polarization ϵ transverse to the D-string is given by [16]

$$\mathcal{A}_B\;\sim\;\frac{\Gamma(-2t)}{\left(\Gamma(1-t)\right)^2}t^2\left(\zeta_1\cdot\epsilon\cdot\zeta_2\right)\;. \tag{17}$$

Using (16) one obtains, once again for vector polarization components of the gravitino being transverse to the D-string, the amplitude (suitably covariantized)

$$\mathcal{A}_F \;\sim\; \frac{\Gamma(-2t)}{\left(\Gamma(1-t)\right)^2}(2t)\left(\bar{u}_1\zeta_2\cdot\gamma\xi_3\cdot k_2+\ldots\right)\;, \tag{18}$$

where, ξ_3 is the polarization vector-spinor of the emitted fermion. (Note we have written only a representative polarization term to illustrate momentum dependence). Strictly speaking, the physical region of interest is for small values of $t \;\sim\; \omega^2$; it is easy to see that in this domain, $|\mathcal{A}_F| \;\sim\; \omega$.

In contrast to standard computations, as is commonly accepted [2], a decaying gas of D-strings does not afford the usual facility of *preparing* an initial state. The standard procedure of *averaging* over initial states for 'unpolarized' initial states must therefore be replaced by a *summation* over all possible initial momentum distributions. If we assume that the gas in question is an ensemble in thermal equilibrium, it follows that the colliding open string Bose and Fermi modes will also 'thermalize' according to their natural statistics, with distributions $\rho_B(\omega, T_B)$ and $\rho_F(\omega, T_F)$ respectively, where, the temperatures T_B, T_F may not be equal. The total decay rate for the D-string with appropriate phase space factors is then given by

$$\Gamma_D \;\approx\; \omega\;\rho_F(T_F)\;\rho_B(T_B)\;\frac{d^3k}{(2\pi)^3}\;. \tag{19}$$

Comparison with our semiclassical analysis, in the limit of $\omega \to 0$, (upto coefficients) leads to, from equations (11) and (12)

$$\begin{aligned} T_B \;&=\; \frac{\omega}{\pi(a+b)} \;\;,\;\; T_F \;=\; \frac{\omega}{3\pi(a+b)} \;\;, a\approx b \\ T_B \;&=\; \frac{\omega}{4\pi b} \;\;,\;\; T_F \;=\; \frac{\omega}{4\pi a} \;\;,\;\; a>b\;. \end{aligned} \tag{20}$$

We have not yet completed the task of computing the temperatures T_B , T_F from the statistical mechanics of massless open string states on the D-string, but hope to report it in the near future. Observe that in the case $a > b$, the results in (12) can also be understood in terms of left-moving and right-moving distributions of open-string modes on the D-string. This, as mentioned, is not manifest in (13). However why the effective fermi temperature is lower valued, is not clear. Also, while our choice of $n = -1$ appears to be an admissible value for s-wave scattering, other solutions also ought to be explored for novel possibilities. The calculation of the exact string amplitude is being pursued to see whether the decay rates match exactly.

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[1] A. Sen, Mod. Phys. Lett. A**10**, 2081(1995); A. Strominger and C. Vafa, Phys. Lett. B.**379**, 99 (1996), hep-th/9601029; G. Horowitz, Report No. gr-qc/9604051 and references therein.

[2] C. Callan and J. Maldacena, Nucl. Phys. B**472**, 591 (1996), hep-th/9602043.

[3] S. R. Das and S. D. Mathur, Phys. Lett. B**375**, 103 (1996), hep-th/9601152; S. R. Das and S. D. Mathur, Nucl. Phys. B**478**, 561 (1996), hep-th/9606185, J. Maldacena and L. Susskind, Nucl. Phys. B **475**, 679 (1996), hep-th/9604042.

[4] A. Dhar, G. Mandal and S. R. Wadia, Phys. Lett. B**388**, 51(1996), hep-th/9605234.

[5] S. W. Hawking and M. M. Taylor-Robinson, Phys. Rev. D. **55**, 7680 (1997), hep-th/9702045.

[6] F. Dowker, D. Kastor and J. Traschen, Report No. hep-th/9702109; R. Emparan, Report No. hep-th/9704204.

[7] S. S. Gubser, Report No. hep-th/9706100.

[8] E. Keski-Vakkuri and P. Kraus, Report No. hep-th/9610045; I. R. Klebanov and S. D. Mathur, Report No. hep-th/9701187; S. P. de Alwis and K. Sato, Report No. hep-th/9611189; S. Das, A. Dasgupta and T. Sarkar, Phys. Rev. D **55**, 7693 (1997), hep-th/9702075.

[9] S. S. Gubser and I. Klebanov, Phys. Rev. Lett. **77** 4491 (1996), hep-th/9609076.

[10] J. Maldacena and A. Strominger, Phys. Rev. D **55**, 861 (1997), hep-th/9609026 and references therein.

[11] W. G. Unruh, Phys. Rev. D **14**, 3251 (1976).

[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, INC. (1980).

[13] S. R. Das, G. Gibbons and S. D. Mathur, Phys. Rev. Lett. **78**, 417 (1997), hep-th/9609052.

[14] M. R. Garousi and R. Myers, Nucl. Phys. B **475**, 193 (1996), hep-th/9603194.

[15] M. B. Green and M. Gutperle, Nucl. Phys. B **476**, 484 (1996), hep-th/9604091.

[16] A. Hashimoto and I. Klebanov, Phys. Lett. B **381** ,437(1996), hep-th/9604065.